

# Spectrum of the gauge Ising model in three dimensions.

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We present a high precision Monte Carlo study of the spectrum of the  $Z_2$  gauge theory in three dimensions in the confining phase. Using state of the art Monte Carlo techniques we are able to accurately determine up to three masses in a single channel. We compare our results with the  $SU(2)$  spectrum and with the prediction of the Isgur-Paton model. Our data strongly support the conjecture that the glueball spectrum is described by some type of flux tube model. We also compare the spectrum with some recent results for the correlation length in the 3d spin Ising model. This analysis sheds light on some nontrivial features of the duality transformation.

## 1. Introduction

The infrared regime of Lattice Gauge Theories (LGT) in the confining phase displays a large degree of universality. The main evidences in favour of this universality are given by the behaviour of the Wilson loop and of the adimensional ratio  $T_c^2/\sigma$  (where  $T_c$  denotes the deconfinement temperature) which are roughly independent from the choice of the gauge group, and show a rather simple dependence on the number of space-time dimensions. Both these behaviours are commonly understood as consequences of the fact that the relevant degrees of freedom in the confining regime are string-like excitations. The phenomenological models which try to keep into account this string-like picture are usually known as “flux-tube” models and turn out to give a very good description of the Wilson loop behaviour (see for instance [1] and references therein). The reason of this success is that in the interquark potential we have a natural scale, the string tension, which allows to define in a rather precise way a large distance (“infrared”) regime in which the adiabatic approximation for the string-like excitations can be trusted. This regime can be reached by considering interquark distances large in units of the string tension.

Besides the Wilson loop and the deconfinement temperature, another important set of physical

observables in LGT is represented by the glueball spectrum. In this case it is less obvious that a string like description could be used to understand the data. However, a string-inspired model exists also for the glueball spectrum: the Isgur Paton model [1] (IP in the following).

For this reason it would be very interesting to test if the same universality (which, as mentioned above, should manifest itself as a substantial independence from the choice of the gauge group) displayed by the Wilson loops also holds for the glueball spectrum. In this case we do not have the equivalent of the interquark distance, *i.e.* a parameter which can be adjusted to select the infrared region: the role of large Wilson loops is played by the higher states of the spectrum which, being localized in larger space regions, are expected to show more clearly a string-like behavior. A major problem in this respect is the lack of precise and reliable data for these higher states. An obvious proposal to overcome this problem is to begin with the (2+1) dimensional case, for which some relevant simplifications occur in the spectrum and a much higher precision can be achieved in the Montecarlo simulations.

Following this suggestion we have studied in [2] the glueball spectrum in the case of the (2+1) Ising gauge model obtaining a high precision estimate of the first 11 states of the spectrum. These can be compared with some results for the (2+1) dimensional  $SU(2)$  model obtained with Monte-

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carlo simulations [3] and with variational techniques [4]. The comparison between the SU(2) and Ising spectra shows that, not only the pattern of the states is the same in the two models, but also the values of the masses (except for the lowest state) are in remarkable agreement. This is a strong evidence in favour of the above mentioned universality, and suggests that the higher states of the glueball spectrum of any LGT, (as it happens for the behaviour of large enough Wilson loops) can be predicted by some relatively simple flux-tube inspired model. We shall give in the next section few general information on the model and on the algorithm that we used (we refer to [2] for further details). Sect 3. is devoted to a discussion of the glueball spectrum and to a comparison with the SU(2) results and with the IP predictions. Finally in the last section we shall make some concluding remark on the duality transformation of the glueball spectrum.

## 2. The Ising gauge model

The Ising gauge model is defined by the action

$$S_{gauge} = -\beta \sum_{n,\mu < \nu} g_{n;\mu\nu} \quad (1)$$

where  $g_{n;\mu\nu}$  are the plaquette variables, defined in terms of the link fields  $g_{n;\mu} \in -1, 1$  as:

$$g_{n;\mu\nu} = g_{n;\mu} g_{n+\mu;\nu} g_{n+\nu;\mu} g_{n;\nu} \quad (2)$$

where  $n \equiv (\vec{x}, t)$  denotes the space-time position of the link and  $\mu$  its direction. For the Ising model, as for the SU(2) model, we cannot define a charge conjugation operator. The glueball states are thus labelled only by their angular momentum  $J$  and by their parity eigenvalue  $P = \pm$ . The standard notation is  $J^P$ . An important simplification due to the fact that we are working in (2+1) dimensions is that in this case all the states with angular momentum different from zero are degenerate in parity. Namely  $J^+$  and  $J^-$  (with  $J \neq 0$ ) must have the same mass. This result holds in the continuum limit. The lattice discretization breaks this degeneration, since in this case the symmetry group is only the  $D_4$  (dihedral) group. In particular it can be shown that the degeneration still holds on the lattice for all

the odd  $J$  states, and is lifted for all the even  $J$  states (see [2] for details). An important test of the whole analysis is to see if the degeneration of the even part of the spectrum is recovered in the continuum limit. There are two other important consequences of the fact the symmetry group is reduced to  $D_4$ . The first one is that only operators with angular momentum  $J \pmod{4}$  can be observed and the second is that all the odd  $J$  states are grouped together in the same irreducible representation of  $D_4$ . This means that we cannot distinguish among them on the basis of the lattice symmetries. Hence in the following we shall denote the states belonging to this channel as  $J = 1/3$  states.

The simulations were performed with a local demon-algorithm in Multi-spin-coding technique. Variational techniques were used to obtain accurate results for the higher states of the spectrum in any given channel. We studied five different values of  $\beta$  in the scaling region, and carefully tested that all the states of the spectrum followed the expected scaling behaviour. As mentioned above we obtained reliable estimates for 11 masses.

## 3. Results and comparison with SU(2) and with the IP model.

A first unambiguous result of the simulation is that the parity degeneration is indeed recovered in the continuum limit also for the even  $J$  sector. Taking into account this degeneration we end up with 8 independent states in the continuum limit. Their masses in the continuum limit (measured in units of the string tension  $\sqrt{\sigma}$ ) are listed in tab.1, where they are compared with the corresponding results for SU(2) obtained in [3] and with the predictions of the IP model.

Looking at tab.1 we see that the biggest discrepancy in the mass values is for the lowest state, which is predicted to be too light in the IP model, and turns out to be very different in the Ising and SU(2) cases. This is due first to the lack of validity of the adiabatic approximation at small scales and second to the fact that in the IP model an “ideal” picture of string (without self repulsion terms) is assumed for the flux tube.

Table 1  
Comparison between the Ising,  $SU(2)$  and IP spectra.

$J^P$	Ising	$SU(2)$	IP
$0^+$	3.08(3)	4.763(31)	2.00
$(0^+)'$	5.80(4)		5.94
$(0^+)''$	7.97(11)		8.35
$2^\pm$	7.98(8)	7.78(10)	6.36
$(2^\pm)'$	9.95(20)		8.76
$0^-$	10.0(5)	9.90(27)	13.82
$(0^-)'$	13.8(6)		15.05
$(1/3)^\pm$	12.7(5)	10.75(50)	8.04

Apart from this state, in the remaining part of the spectrum we immediately see an impressive agreement between the Ising and  $SU(2)$  spectra. This agreement is further improved by looking at the excited states in the  $(0^+)$  channel for the  $SU(2)$  model. In [4] a variational estimate for these masses can be found (up to our knowledge no Montecarlo estimate exists for them). In tab.2 we compare these values with the Ising ones. While the two sets of excited states disagree if measured in units of  $\sqrt{\sigma}$ , they agree if measured in units of  $0^+$ . Moreover a better and better agreement is observed if ratios of higher masses are considered.

Table 2  
The  $0^+$  channel.

ratio	Ising	$SU(2)$
$(0^+)'/0^+$	1.88(2)	1.77(2)
$(0^+)''/0^+$	2.59(4)	2.50(5)
$(0^+)''/(0^+)'$	1.37(4)	1.41(4)

We can conclude from these data that the qualitative features of the glueball spectrum are largely independent from the gauge group and well described by a flux tube effective model. While the higher states of the spectrum show a remarkable independence from the gauge group,

for the lowest state the flux tube picture breaks down and the gauge group becomes important. The IP model, which is the simplest possible realization of such a flux tube, seems able to catch (at least at a qualitative level) some of the relevant features of the glueball spectrum.

#### 4. Duality.

Another important reason of interest in the gauge Ising model is that it is related by duality to the ordinary 3D spin Ising model. As a consequence, one expects the glueball spectrum to be mapped in the spectrum of massive excitations of the spin model. However this mapping has some non trivial features. While the lowest state  $0^+$  is mapped into the (inverse) correlation length of the spin model (see [2,5] for details), it is not clear which are the dual partners of the higher states of the spectrum. In fact one naively expects that the spin Ising model should be described in the scaling region by a  $\phi^4$  theory, whose spectrum however contains just one state. As a matter of fact the higher states of the gauge Ising model correspond to “disorder” variables in the spin Ising model which are *non-local* with respect to the “order” variable (the  $\phi$  field). The overlap between these mutually non-local observables becomes vanishingly small in the continuum limit (even if it is non-zero in any finite lattice) thus explaining the difference between the two dual spectra. A detailed comparison of correlation functions in the Ising spin model and in the  $\phi^4$  theory seems to confirm this picture [6].

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